Sirius University of Science and Technology
Sirius Mathematics Center

International Conference

Analysis Days in Sirius

25–29 October 2021

Program and Abstracts

Sochi, 2021

Program and Organizing Committee:

Alexander Borichev

Konstantin Fedorovskiy

Vladimir Lysov

Pavel Mozolyako

Petr Paramonov

Scientific Secretary:

Elijah Lopatin

Analysis Days in Sirius

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The conference is organised in cooperation with the Moscow Center of Fundamental and Applied Mathematics.

The Conference is held at the Sirius Mathematics Center. The organizers are grateful to the staff of the SMC and, in particular, to Alexei Shchuplev for their help and support.

International Conference “Analysis Days in Sirius”:
https://mathnet.ru/php/conference.phtml?confid=1995

Speakers:

- Alexander Aptekarev, *Keldysh Institute of Applied Mathematics of RAS, Moscow, Russia*
- Nicola Arcozzi, *University of Bologna, Italy*
- Astamur Bagapsh, *Bauman Moscow State Technical University & Federal Research Center 'Computer Science and Control' of RAS, Russia*
- Anton Baranov, *Saint Petersburg State University, Russia*
- Valery Beloshapka, *Moscow State University & Moscow Center of Fundamental and Applied Mathematics, Russia*
- Yurii Belov, *Saint Petersburg State University, Russia*
- Sergei Bezrodnykh, *Federal Research Center 'Computer Science and Control' of RAS & Peoples' Friendship University, Moscow, Russia*
- Andrei Bogatyrev, *Institute of Numerical Mathematics of RAS & Moscow State University & Moscow Center of Fundamental and Applied Mathematics, Russia*
- Andrei Domrin, *Moscow State University & Moscow Center of Fundamental and Applied Mathematics, Russia*
- Alexander Dyachenko, *Keldysh Institute of Applied Mathematics of RAS, Moscow, Russia*
- Nicholas Hatzizisis, *Moscow State University, Russia & University of Crete, Greece*
- Dmitry Khavinson, *University of South Florida, Tampa, USA*
- Alexander Komlov, *Steklov Mathematical Institute of RAS, Moscow, Russia*
- Maksim Mazalov, *National Research University 'Moscow Power Engineering Institute' in Smolensk, Russia*
- Alexander Mkrtchan, *Institute of Mathematics of NAS of RA, Erevan, Armenia & Siberian Federal University, Krasnoyarsk, Russia*
- Joaquim Ortega, *University of Barcelona, Spain*
- Nikolay Osipov, *Saint Petersburg Department of Steklov Mathematical Institute of RAS, Russia*
- Roman Pal'velev, *Moscow State University, Russia*
- Roman Romanov, *Saint Petersburg State University, Russia*
- Maria Stepanova, *Moscow State University, Russia*
- Sergei Suetin, *Steklov Mathematical Institute of RAS, Moscow, Russia*
- Pascal Thomas, *University of Toulouse, France*

- Xavier Tolsa, *ICREA & Universitat Autonoma de Barcelona, Spain*
- Mikhail Tyaglov, *Shanghai Jiao Tong University, China*
- Alexander Ulanovskii, *University of Stavanger, Norway*
- Alexander Volberg, *Michigan State University, East Lansing, USA*
- Rachid Zarouf, *Aix-Marseille Université, France*

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- Pavel Mozolyako, *Saint Petersburg State University, Russia*
- Petr Paramonov, *Moscow State University, Russia*

Scientific Secretary:

- Elijah Lopatin, *Steklov Mathematical Institute of RAS, Moscow, Russia*

CONFERENCE PROGRAM

MONDAY 25 OCTOBER

Chairman: Konstantin Fedorovskiy

10^{00} — 10^{45} Yurii Belov (Saint Petersburg State University; Russia). *On the chain structure of de Branges spaces.*

10^{50} — 11^{35} Joaquim Ortega (University of Barcelona; Spain). *Idempotent Fourier multipliers acting contractively on H^p space.*

BREAK

Chairman: Andrei Bogatyrev

11^{55} — 12^{40} Mikhail Tyaglov (Shanghai Jiao Tong University; China). *From Krawtchouk to Hahn via Jacobi and some other applications of differential operators with polynomial coefficients.*

12^{45} — 13^{30} Sergei Suetin (Steklov Mathematical Institute of RAS; Russia). *A direct proof of Stahl's theorem for a generic class of algebraic functions.*

LUNCH

Chairman: Vladimir Lysov

15^{00} — 15^{45} Astamur Bagapsh (Bauman Moscow State Technical University and Federal Research Center 'Computer Science and Control' of RAS; Russia). *Mappings by solutions of strongly elliptic systems.*

15^{50} — 16^{35} Sergei Bezrodnykh (Federal Research Center 'Computer Science and Control' of RAS and Peoples' Friendship University; Russia). *The Lauricella function and its relationship with other hypergeometric functions.*

BREAK

Chairman: Yurii Belov

16^{55} — 17^{40} Nicola Arcozzi (University of Bologna; Italy). *Some problems in discrete analysis with applications to function spaces.*

17^{45} — 18^{30} Andrei Domrin (Moscow State University and Moscow Center of Fundamental and Applied Mathematics; Russia). *Monodromy-free operators and holomorphic solutions of soliton equations.*

TUESDAY 26 OCTOBER

Chairman: Andrei Domrin

10^{00} — 10^{45} Aleksandr Komlov (Steklov Mathematical Institute of RAS; Russia). *Constructive reconstruction of values of an algebraic function via polynomial Hermite–Padé m-systems.*

10^{50} — 11^{35} Valerii Beloshapka (Moscow State University and Moscow Center of Fundamental and Applied Mathematics; Russia). *Quasi-homogeneous model CR manifolds.*

BREAK

Chairman: Pavel Mozolyako

11^{55} — 12^{40} Nikolay Osipov (St. Petersburg Department of Steklov Mathematical Institute of RAS; Russia). *Bellman function for the Gundy theorem.*

12^{45} — 13^{30} Rachid Zarouf (Aix-Marseille Université; France). *Schäffer's conjecture, Fourier coefficients of Blaschke products and Jacobi polynomials with first varying parameter.*

LUNCH

Chairman: Valery Beloshapka

15^{00} — 15^{45} Maksim Mazalov (National Research University 'Moscow Power Engineering Institute' in Smolensk; Russia). *Bianalytic functions of Hölder classes in Jordan domains with nonanalytic boundaries.*

15^{50} — 16^{35} Alexander Ulanovskii (University of Stavanger; Norway). *On geometry of the unit ball of Paley–Wiener space over two intervals.*

BREAK

Chairman: Alexander Borichev

16^{55} — 17^{55} Alexander Volberg (Michigan State University; USA). *Strange property of positive measures and bi-linear estimates on multi-trees.*

18^{00} — 19^{00} Dmitry Khavinson (University of South Florida; USA). *The classification problem for arclength null quadrature domains.*

THURSDAY 28 OCTOBER

Chairman: Petr Paramonov

09^{45} — 10^{30} Roman Romanov (Saint Petersburg State University; Russia). *On zeroes and poles of Helson zeta function.*

10^{35} — 11^{20} Xavier Tolsa (ICREA and Universitat Autonoma de Barcelona; Spain). *The measures with L^2-bounded Riesz transform and the Painlevé problem for Lipschitz harmonic functions.*

BREAK

Chairman: Konstantin Fedorovskiy

11^{40} — 12^{25} Pascal Thomas (University of Toulouse; France). *Invertibility threshold for Nevanlinna quotient algebras.*

12^{30} — 13^{30} Alexander Aptekarev (Keldysh Institute of Applied Mathematics of RAS; Russia). *Recurrence relations and asymptotics of colored Jones polynomials.*

LUNCH

OPEN PROBLEM SESSIONS

15^{00} — 16^{35} OPS 1: *"Approximation, interpolation and sampling in spaces of analytic functions."* Coordinators: Yu. Belov, K. Fedorovskiy, P. Paramonov.

15^{00} — 16^{35} OPS 2: *"Spectral theory of difference operators: weak and strong asymptotics."* Coordinators: A. Aptekarev, V. Lysov.

BREAK

16^{55} — 18^{30} OPS 3: *"Open questions of multivariable complex analysis."* Coordinators: V. Beloshapka, A. Domrin.

16^{55} — 18^{30} OPS 4: *"Constructive approximations and analytic continuation on Riemann surfaces."* Coordinators: A. Komlov, S. Suetin.

18^{45} CONFERENCE PARTY

FRIDAY 29 OCTOBER

Chairman: Mikhail Tyaglov

09^{45} — 10^{30} Alexander Dyachenko (Keldysh Institute of Applied Mathematics of RAS; Russia). *On algebraic properties of classical multiple orthogonal polynomials of discrete variable.*

10^{35} — 11^{20} Alexander Mkrtchan (Institute of Mathematics of NAS of RA; Armenia and Siberian Federal University; Russia). *Analytic continuation of multiple power series by means of coefficients of interpolation.*

BREAK

Chairman: Alexander Aptekarev

11^{40} — 12^{25} Andrei Bogatyrev (Institute of Numerical Mathematics of RAS and Moscow State University and Moscow Center of Fundamental and Applied Mathematics; Russia). *Graphs describing conformal structure and their degeneration.*

12^{30} — 13^{30} Anton Baranov (Saint Petersburg State University; Russia). *Hilbert spaces of Cauchy transforms.*

LUNCH

Abstracts of the talks

Recurrence relations and asymptotics of colored Jones polynomials

Alexander Aptekarev

Keldysh Institute of Applied Mathematics

We consider q-difference relations for the colored Jones polynomials. These sequences of polynomials are invariants for the knots and their asymptotics plays an important role in the famous volume conjecture for the complement of the knot to the 3D sphere. We give an introduction to the theory of hyperbolic volume of the knot complements and study the asymptotics of the solutions of the q-recurrence equations of higher order.

Some problems in discrete analysis with applications to function spaces

Nicola Arcozzi

University of Bologna

We survey, with motivation, some problems in discrete potential theory whose solution would have consequences in the analysis of holomorphic function spaces. We will mention open problems concerning the conformally invariant theory for the Dirichlet space, the Drury–Arveson space, polydiscs, et cetera.

Mappings by solutions of strongly elliptic systems

Astamur Bagapsh

Bauman Moscow State Technical University

Boundary properties and univalence of solutions of strongly elliptic systems of 2nd order on the plane are discussed. The most studied are the mappings by complex-valued harmonic functions. Such mappings can often be “lifted” to mappings of surfaces. We give an example of a mapping by the Poisson kernel for a plane Lame system, which can also be naturally associated with a mapping of surfaces, but, in contrast to the harmonic case, the method of “lifting” a mapping in a more general situation is still unclear.

We consider the mappings by solutions of strongly elliptic system

$$\left(A\frac{\partial^2}{\partial x^2} + 2B\frac{\partial^2}{\partial x \partial y} + C\frac{\partial^2}{\partial y^2}\right)\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

on real function $u(x, y)$ and $v(x, y)$ on the complex plane with real 2×2 matrices A, B, C. Strong ellipticity means that $\det(\alpha A + 2\beta B + \gamma C) \neq 0$ for $\beta^2 - \alpha\gamma < 0$. Such a system can be reduced to the canonical form

$$\mathcal{L}_{\tau,\sigma} f = (\partial\overline{\partial} + \tau\partial^2) f + \sigma(\tau\partial\overline{\partial} + \partial^2)\overline{f} = 0 \tag{1}$$

with a complex-valued function f and the parameters $\tau, \sigma \in [0, 1)$. The classical example is the Laplace equation ($\tau = \sigma = 0$) which is satisfied by harmonic functions.

One of the remarkable properties of the harmonic mappings is the ability to send points to curves and curves to points, which is in contrast to holomorphic functions, the solutions of the 1st order Cauchy–Riemann system. The same properties are demonstrated by solutions of the strongly elliptic systems. It can be shown that the angular limits of such a solution f in a jump point of its boundary values form a smooth curve

$$\Gamma = \{w = p_{\tau,\sigma}(\theta, \alpha)\varphi_+(\theta) + (\mathcal{I} - p_{\tau,\sigma}(\theta, \alpha))\varphi_-(\theta),\ \alpha \in (0, \pi)\},$$

where θ is the tangent angle, $\varphi_\pm(\theta)$ are one-side limits of f along the boundary, and

$$p_{\tau,\sigma}(\theta, \alpha) = \frac{\alpha}{\pi}\mathcal{I} + \frac{1}{2\pi i} \log \frac{1 + \tau e^{-2i\theta}}{1 + \tau e^{2i(\alpha-\theta)}} \left(\mathcal{I} + \frac{\sigma}{\tau}\mathcal{C}\right)$$

with $\mathcal{I}$ being the identity and $\mathcal{C}$ being the complex conjugation operator. In addition, the Poisson kernel for solution to a strongly elliptic system in a disk maps its boundary without a single point to the origin.

The mentioned properties usually have to be considered in connection with the question of univalence of the mappings by solutions of the systems. In recent decades, the attempts to generalize the classical theorem of Rado, Kneser and Choquet, which gives the sufficient condition for univalence, have shown that this theorem fails for planar Lame systems, which is the case of the equation (1) with $\tau = 0$.

It is interesting to consider the mapping of the unit disk by the Poisson kernel for the Lame system with singularity at the point 1:

$$P_\sigma(z) = \frac{1}{2\pi}(1 - |z|^2)\left(\frac{1}{|1 - z|^2} + \sigma\frac{2 - z}{(1 - z)^2}\right).$$

This mapping in not univalent but it can be naturally associated with a one-to-one mapping of the elliptic cone to the surface of the revolution of hyperbola. It is well known that the harmonic mappings can also be associated with mappings onto the minimal surfaces, but the mappings of the plane "lifted" in such a way are univalent themselves. The given example of the Poisson kernel for the Lame system shows that the solutions to strongly elliptic systems are probably more naturally related to one-to-one mappings of surfaces rather than plane domains.

Hilbert spaces of Cauchy transforms

Anton Baranov

Saint Petersburg State University

We consider the spaces of functions which can be represented as the Cauchy integrals with L^2 data with respect to some fixed measure in the complex plane. These are Reproducing Kernel Hilbert spaces of functions analytic outside the support of the measure. They naturally appear in functional models for rank one perturbations of normal operators.

In the talk we concentrate on the case of discrete measures and discuss several properties of associated spaces of entire functions, such as:

1. Sets of uniqueness;

2. Existence of orthogonal bases and Riesz bases of reproducing kernels;

3. Structure of nearly invariant (i.e., division invariant) subspaces;

4. Existence of nearly invariant subspaces of finite codimension and the domain of multiplication by the independent variable;

5. Localization of zeros.

Quasi-homogeneous model CR manifolds

Valery Beloshapka

Moscow State University

Smooth real submanifolds of complex spaces and their holomorphic mappings are a subject of study in CR geometry. Both analytical and geometrical methods are used here. We are going to discuss recent advances in the analytical approach, which is connected with an arbitrary choice of the weights in the complex tangent space.

The analytic technique of formal power series and homological operators, which goes back to H. Poincare, is used effectively in analysis and geometry. Its applications to CR geometry (the model surface method) keep developing, and at the same time connections with other fields of mathematics are coming to light (the Tanaka theory of graded Lie algebras, holomorphic dynamics, commutative algebra). In this approach we are focused on the model surfaces — the surfaces, which are graphs of polynomials of a special type and which are the most holomorphically symmetrical objects. In recent papers the basic construction became more flexible and included a wide class of CR manifolds in the sphere of its applicability. In 2020, the corresponding construction, which is applicable to an arbitrary germ of generic CR manifold of finite Bloom–Graham type, was described. Weights and gradings of commutative rings and Lie algebras began to be used a long time ago. But in the classical version the variables in the complex tangent space had equal weights. On the other hand, there are a great deal of interesting examples, where *different* weights in the complex tangent are used. In 2021, the "weighted" version of the model surface method was described. This version is adapted to operate with arbitrary CR manifolds of finite type. Here a modification of the type of a germ, described in 2018, is used (Bloom–Graham–Stepanova type). Within the framework of the new version the main statements are formulated and proved (maximal symmetricity, the structure of the Lie algebra of infinitesimal holomorphic automorphisms, etc.). New open questions arose.

This weighted approach allowed us to conceptualize the available examples from a unified point of view. In particular, the new theory includes the hypersurfaces of J. Winkelmann, A. Loboda, A. Labovskii, B. Kruglikov, I. Zelenko and their generalizations. These hypersurfaces were described earlier as the most symmetrical in different classes.

On the chain structure of de Branges spaces

Yurii Belov

Saint Petersburg State University

It is well known that any measure μ (with $\int(1+x^2)^{-1}d\mu(x) < \infty$) on the real line generates a chain of Hilbert spaces of entire functions (de Branges spaces). These spaces are isometrically embedded in $L^2(\mu)$. We study the indivisible intervals and the stability of exponential type in the chains of de Branges subspaces in terms of the spectral measure.

The talk is based on joint work with Alexander Borichev.

The Lauricella function and its relationship with other hypergeometric functions

Sergei Bezrodnykh

Federal Research Center 'Computer Science and Control' & RUDN University

The Lauricella function $F_D^{(N)}(\mathbf{a}; b,\, c;\, \mathbf{z})$ of the variable $\mathbf{z} := (z_1, \ldots, z_N) \in \mathbb{C}^N$, depends of the parameters $\mathbf{a} := (a_1, \ldots, a_N) \in \mathbb{C}^N$, $b,\, c, \in \mathbb{C}$ and is defined as the following N–multiple hypergeometric series:

$$F_D^{(N)}(\mathbf{a}; b,\, c;\, \mathbf{z}) := \sum_{|\mathbf{k}|=0}^{\infty} \frac{(b)_{|\mathbf{k}|}(a_1)_{k_1}\cdots(a_N)_{k_N}}{(c)_{|\mathbf{k}|}k_1!\cdots k_N!}\, z_1^{k_1}\cdots z_N^{k_N}\,, \tag{1}$$

converging in the unit polydisk $\mathbb{U}^N := \left\{\mathbf{z} \in \mathbb{C}^N\colon |z_j| < 1,\, j = \overline{1,N}\right\}$, where $(a)_k$ is the Pochhammer symbol; $\mathbf{k} := (k_1, \ldots, k_N)$ is the multi-index. The series (1) satisfies the following system of N linear partial differential equations of the second order with respect to variables z_j, see [1], [2]:

$$z_j(1-z_j)\partial^2_{jj}u + (1-z_j)\sum_{k=1}^{N}{}' z_k\partial^2_{jk}u + \Big[c - (1+a_j+b)z_j\Big]\partial_j u - a_j\sum_{k=1}^{N}{}' z_k\partial_k u - a_j b\, u = 0; \tag{2}$$

here $j = \overline{1,N}$, and the "stroke" over the sum means that the summation is carried out by $k \neq j$. It is known [1], [2] that the holomorphic rank of this system equals $N+1$, i.e. in the neighborhood of a non-singular point from $\mathbb{C}^N$, the system has $N+1$ linearly independent solutions, and its general solution depends on $N+1$ constant.

The talk is focused on study of the relationship of the Lauricella function theory and the corresponding system of equations (2) with the theory of the Horn hypergeometric functions [3], [4] and the Gelfand–Kapranov–Zelevinsky (GKZ) functions [5]. The issue of the analytical continuation of the series (1) is also considered, which is one of the main ones also for the Horn and the GKZ functions.

One of the variants of the analytic continuation theorem is the following statement [6].
Theorem. *Suppose that none of the numbers* $b - \sum_{s=1}^{j} a_s$, $j = \overline{1,N}$, *is an integer. Then the analytic continuation of the series* (1) *into the domain*

$$\mathbb{V}^N := \left\{ \mathbf{z} : |z_1| > \cdots > |z_N| > 1;\ |\arg(-z_j)| < \pi, j = \overline{1,N} \right\},$$

is given by the formula

$$F_D^{(N)}(\mathbf{a};\, b,\, c;\, \mathbf{z}) \;=\; \sum\nolimits_{j=0}^{N} B_j \mathscr{U}_j^{(\infty)}(\mathbf{a};\, b,\, c;\, \mathbf{z}), \tag{3}$$

where the functions $\mathscr{U}_0^{(\infty)}$, $\mathscr{U}_j^{(\infty)}$ *are defined by*

$$\mathscr{U}_0^{(\infty)}(\mathbf{a};\, b,\, c;\, \mathbf{z}) = \left(\prod\nolimits_{l=1}^{N} (-z_l)^{-a_l}\right) F_D^{(N)}\left(\mathbf{a};\, 1+|\mathbf{a}|-c,\, 1+|\mathbf{a}|-b;\, \mathbf{z}^{-1}\right), \tag{4}$$

$$\begin{aligned} \mathscr{U}_j^{(\infty)}(\mathbf{a};\, b,\, c;\, \mathbf{z}) &= (-z_j)^{|\mathbf{a}_{1,j-1}|-b} \left(\prod_{l=1}^{j-1} (-z_l)^{-a_l}\right) \times \\ &\times\ G^{(N,j)}\Big(\mathbf{h}_j; b - |\mathbf{a}_{1,j-1}|, 1 - |\mathbf{a}_{1,j}| + b; \mathscr{Y}_j(\mathbf{z}^{-1})\Big), \qquad j = \overline{1,N}. \end{aligned} \tag{5}$$

Here

$$G^{(N,j)}\big(\mathbf{a};\, b,\, c;\, \mathbf{z}\big) \;:=\; \sum\nolimits_{|\mathbf{k}|=0}^{\infty} \frac{(b)_{|\mathbf{k}_j|}\, (\mathbf{a})_{\mathbf{k}}}{(c)_{|\mathbf{k}_j|}\, \mathbf{k}!}\, \mathbf{z}^{\mathbf{k}}, \tag{6}$$

$\mathbf{h}_j := (a_1, \ldots, a_{j-1}, 1-c+b, a_{j+1}, \ldots a_N)$, $|\mathbf{a}_{1,j}| := \sum_{k=1}^{j} a_k$, $|\mathbf{a}| := |\mathbf{a}_{1,N}|$,

$$\mathbf{z}^{-1} := \left(z_1^{-1}, \ldots, z_N^{-1}\right), \qquad \mathscr{Y}_j(\mathbf{z}) := \left(z_1/z_j, \ldots, z_{j-1}/z_j, z_j, z_j/z_{j+1}, \ldots, z_j/z_N\right),$$

and the coefficients B_j *have the form*

$$B_0 = \frac{\Gamma(c)\,\Gamma\big(b-|\mathbf{a}|\big)}{\Gamma(b)\,\Gamma\big(c-|\mathbf{a}|\big)}, \qquad B_j = \frac{\Gamma(c)\,\Gamma\big(b-|\mathbf{a}_{1,j-1}|\big)\Gamma\big(|\mathbf{a}_{1,j}|-b\big)}{\Gamma(a_j)\Gamma(b)\Gamma(c-b)}, \quad j = \overline{1,N}. \tag{7}$$

The functions (4), (5) *are linearly independent solutions of the Lauricella system of differential equations* (2) *in the domain* $\mathbb{V}^N$.

The work is supported by the Ministry of Science and Higher Education of the Russian Federation: agreement no. 075-03-2020-223/3 (FSSF-2020-0018).

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[2] Exton H., *Multiple hypergeometric functions and application.* J. Willey & Sons, New York, 1976.

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[5] Gel'fand I. M., Graev M. I., Retakh V. S., "General hypergeometric systems of equations and series of hypergeometric type." *Russ. Math. Surv.,* 47(4): 1–88, 1992.
[6] Bezrodnykh S. I., "The Lauricella hypergeometric function $F_D^{(N)}$, the Riemann–Hilbert problem, and some applications." *Russ. Math. Surv.,* 73(6): 941–1031, 2018.

Graphs describing conformal structure and their degeneration

Andrei Bogatyrev

Institute of Numerical Mathematics & Moscow State University

Riemann surfaces do not necessarily appear as algebraic curves. For some applications other representations are much more convenient. For instance, one can glue a surface of some standard pieces of complex plane: half planes, triangles, rectangles, (half-) stripes, etc. The scars remaining after this surgery make up a graph embedded into the surface. The idea to represent complex structures and even more fine objects on surfaces by embedded weighted graphs was possibly introduced by Felix Klein. Contemporary examples of this kind include Dessins d'Enfants (A. Grothendieck), Ribbon graphs (M. Kontsevich), Triangulated surfaces (G. Shabat, V. Voevodsky), Flat surfaces (A. Zorich) and Chebyshev Ansatz (AB). Pictorial technique proved to be extremely useful in the study of meso-scale geometry of moduli spaces of curves.

In this talk we discuss the pictorial technique arising in uniform rational approximation problems and consider two typical degenerations of graphs: (i) junction of two branchpoints of a surface and (ii) merging of two zeros of the distinguished abelian differential.

References

[1] Bogatyrev A., "Coordinate spaces of graphs: approaching interior faces." arXiv:2009.11822, 2020.

Monodromy-free operators and holomorphic solutions of soliton equations

Andrei Domrin

Moscow State University

A Schrödinger operator $L = \partial_x^2 + u(x)$ with potential $u(x)$ meromorphic in a domain $D \subset \mathbb{C}$, is said to be *mondromy-free* if all solutions $\varphi(x)$ of the equation $L\varphi = z^2\varphi$ are meromorphic in D for every $z \in \mathbb{C}$. Description of such operators for various classes of meromorphic functions $u(x)$ is a traditional question of spectral theory [1] with many recent applications to the theory of orthogonal polynomials and rational solutions of Painlevé-type equations (surveyed in [2]) and finite-gap solutions of soliton equations (surveyed in [3]).

We are interested in the relation of this notion to holomorphic solutions of soliton equations. It is known [4] that a potential $u(x)$ of the Weierstrass class (which consists of all elliptic functions, rational functions of e^{ax} bounded at ∞, and rational functions of x vanishing at ∞) is monodromy-free if and only if it is a stationary (independent of t) solution of some isospectral deformation equation $L_t = [A, L]$, where the operator $A = \partial_x^m + v_1\partial_x^{m-2} + \cdots + v_{m-1}$ is such that the commutator $[A, L]$ is a differential operator of degree 0, that is, multiplication by a function (for every odd $m > 1$, this determines $v_1, \ldots, v_{m-1}$ almost uniquely as polynomials in u and its derivatives with respect to x; for example, when $m = 3$, one can choose $v_1 = 3u/2$ and $v_2 = 3u'/4$, which gives rise to the Korteweg–de Vries equation $4u_t = u_{xxx} + 6uu_x$). Using [5], we establish the following generalization of the necessity part.

Theorem 1. *Suppose that $u_0(x)$ is meromorphic in D and the Cauchy problem $u(x, 0) = u_0(x)$ for the equation $L_t = [A, L]$ has a holomorphic solution $u(x, t)$ on $\{|x - x_0| < \varepsilon_1, |t| < \varepsilon_2\}$ for some $x_0 \in D$, $\varepsilon_1, \varepsilon_2 > 0$. Then the operator $L_0 = \partial_x^2 + u_0(x)$ is monodromy-free.*

The sufficiency is absent in general (a monodromy-free operator need not be admissible initial data for the Cauchy problem), and this phenomenon can be characterized quantitatively.

There is a complete analogue of Theorem 1 for Dirac (Zakharov-Shabat) operators and the corresponding equations NLS and mKdV as well as for higher-order operators and systems. We also have a series of local isomonodromy criteria, which are similar to the well-known conditions in [1]. Here is a typical result.

Theorem 2. *Suppose that $u(x), v(x)$ have simple poles at $x = x_0$. All solutions $\varphi(x), \psi(x)$ of the system $\varphi' = z\varphi + u\psi$, $\psi' = v\varphi - z\psi$ are meromorphic at x_0 for every $z \in \mathbb{C}$ if and only if there is a positive integer m such that $u_0v_0 = m^2$ and $u_kv_0 = (-1)^k u_0v_k$ for $k = 1, \ldots, 2m$, where u_k (v_k) is the coefficient of $(x - x_0)^{k-1}$ in the Laurent series expansion of $u(x)$ ($v(x)$) at x_0.*

This work was done with the support of RFBR (grant no. 19-01-00474).

References

[1] Duistermaat J. J., Grünbaum F. A., “Differential equations in the spectral parameter.” *Commun. Math. Phys.,* 103: 177–240, 1986.

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On algebraic properties of classical multiple orthogonal polynomials of discrete variable

Alexander Dyachenko

Keldysh Institute of Applied Mathematics

There are many ways to define multiple orthogonal polynomials with respect to the classical continuous weights. Bearing in mind a deep connection between the classical discrete and continuous orthogonality, we adapt to the discrete case the approach of [1–3] preserving a kind of the Rodrigues formula. Our work [4] introduced a new class of polynomials of multiple orthogonality with respect to the product of classical discrete weights on integer lattices with noninteger shifts.

This talk is devoted to further progress in this direction for the case of two measures. In particular, we derive a third-order linear difference equation, our polynomials are satisfied. Furthermore, we obtain coefficients for the four-term recurrence relations connecting polynomials with indices on "diagonals" (including the "step line"). The initial conditions for these relations are presented by semi-classical extensions of discrete orthogonal polynomials studied in [5–7]: they are of a particular interest since the corresponding recurrence coefficients satisfy various Painlevé equations.

This is a joint work with Vladimir Lysov.

References

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The classification problem for arclength null quadrature domains

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A planar domain is referred to as an arclength null quadrature domain if the integral along the boundary of any analytic in the domain function, representable by the Cauchy integral of its boundary values (Smirnov class E^1), vanishes. Obviously, all such domains must be unbounded. We prove the existence of a "roof function" (a positive harmonic function whose gradient coincides with the inward pointing unit normal along the boundary) for arclength null quadrature domains having finitely many boundary components. This bridges a gap toward classification of arclength null quadrature domains by removing an a priori assumption from previous classification results, in particular, it completes the theorem of Khavinson–Lundberg–Teodorescu from 2012 that established that domains allowing "roof functions" are arclength null quadrature domains. This result also strengthens an existing connection to free boundary problems for Laplace's equation and the hollow vortex problem in fluid dynamics. The proof is based on the techniques originated in classical works of Ahlfors, Carleman and Denjoy. We shall also discuss the current status of the classification problem for arclength null quadrature domains.

This is a 2021 joint work with Erik Lundberg.

Constructive reconstruction of values of an algebraic function via polynomial Hermite–Padé m-system

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Let f_0 be a given germ of some algebraic function f of degree $m+1$. We consider the following problem: how to reconstruct constructively the values of f in "as large a region as possible" on its Riemann surface $\mathfrak{R}$?

We introduce the polynomial Hermite–Padé m-system, which, in our case, is constructed from the tuple of the germs $[1, f_0, f_0^2, \dots, f_0^m]$. This system includes the Hermite–Padé polynomials of the first and of the second type.

We show how to reconstruct the values of f on the first m sheets of the so-called Nuttall partition of the Riemann surface $\mathfrak{R}$ via the polynomial Hermite–Padé m-system.

Bianalytic functions of Hölder classes in Jordan domains with nonanalytic boundaries

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Bianalytic functions are solutions of the equation $\overline{\partial}^2 f = 0$ on open subsets of the complex plane $\mathbb{C}$ where $\overline{\partial} = (\partial/\partial x + i\partial/\partial y)/2$ is the Cauchy–Riemann operator. It can be readily verified that every function bianalytic in a given open set $G \subset \mathbb{C}$ is uniquely represented in the form $f(z) = f_0(z) + \overline{z} f_1(z)$ where $\overline{z} = x - iy$, and f_0 and f_1 are holomorphic functions in G.

Let $G \subset \mathbb{C}$ be a Jordan domain, Γ be its boundary, $\gamma \subset \Gamma$ be a boundary arc, and f be a bianalytic function in G belonging to the class $C(\overline{G}) = C(G \cup \Gamma)$. We denote by $A_2(\overline{G})$ the class of all such functions. We are dealing with the questions of the $A_2(\overline{G})$ functions boundary behavior. If γ is an analytic arc, the situation is not difficult. There exists a holomorphic function A in some neighborhood of γ such that $\overline{z} = A(z)$ on γ (this function A is called the Schwarz function of γ). Given a function $f \in A_2(\overline{G})$ we consider its so-called associated function

$f_\gamma(z) = f_0(z) + A(z)f_1(z)$, which is holomorphic in a one-sided neighborhood of γ lying in G, and it is clear that $f = f_\gamma$ on γ.

In the case of domains with nonanalytic boundaries, the situation is much more difficult and meaningful. A wide class of domains for which the homogeneous Dirichlet problem for bianalytic functions has nontrivial solutions is the class of Nevanlinna domains. It turned out that the boundaries of Nevanlinna domains can be nowhere analytic, it can be a fractal set and, moreover, it can have any Hausdorff dimension from 1 to 2.

At the same time, there exists a Jordan domain G bounded by infinitely smooth curve Γ and having the following property. Let $G' \subset G$ be an arbitrary Jordan domain such that its boundary arc γ is a part of Γ, and let f be a bianalytic function in G'. If f has zero angular boundary values on some subset of γ of positive length, then $f \equiv 0$ in G'. It is clear that Γ cannot intersect any analytic arc by a set of positive length.

Now we consider another boundary behavior effect for bianalytic functions. Even in domains with infinitely smooth boundaries, boundary uniqueness conditions may depend on Hölder class, to which the functions belong. Namely, we prove [1] that for any α and β such that $0 < \alpha < \beta < 1$, there exists a Jordan domain $G = G(\alpha, \beta)$ possessing the following two properties: (i) there exists a non-constant function of the class $\mathrm{Lip}_\alpha(\overline{G})$ which is bianalytic in G and vanishes identically on the boundary ∂G of G; (ii) every arc containing in ∂G is a uniqueness set for functions bianalytic in G and belonging to the class $\mathrm{Lip}_\beta(\overline{G})$. The construction of such G uses Wolff–Denjoy series, estimates of harmonic measure and infinite products of unbounded Nevanlinna characteristic.

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Analytic continuation of multiple power series by means of coefficients interpolation

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One way to explore an analytic function is to expand it as a power series. The coefficients of a power series expansion contain the information of its analytic continuation. One possible approach to treat analytic continuation problem is to interpolate the coefficients by values $\varphi(k)$ of an entire function $\varphi(z)$ at the natural numbers $k \in \mathbb{N}$.

In case of a one variable power series, Arakelian gave a criterion for a given arc of a unit circle to be an arc of regularity in terms of the indicator of the interpolating entire function. Pólya found conditions for analytic continuability of a series to the whole complex plane except for some boundary arc.

We give conditions for analytic extension of a multiple power series to a sectorial domain. Consider the multiple power series

$$f(z) = \sum_{k \in \mathbb{N}^n} f_k z^k. \tag{1}$$

Following V. Ivanov we introduce the set which implicitly carries information on the growth indicator of an entire function $\varphi(z) \in \mathcal{O}(\mathbb{C}^n)$:

$$T_\varphi(\theta) = \{\nu \in \mathbb{R}^n : \ln|\varphi(re^{i\theta})| \le \nu_1 r_1 + ... + \nu_n r_n + C_{\nu,\theta}\},$$

where the inequality is satisfied for any $r \in \mathbb{R}^n_+$ with some constant $C_{\nu.\theta}$.

Denote

$$T_\varphi := \bigcap_{\theta_j = \pm\frac{\pi}{2}} T_\varphi(\theta_1, ..., \theta_n),$$

$$\mathcal{M}_\varphi := \{\nu \in [0,\pi)^n : \ \nu + \varepsilon \in T_\varphi, \ \ \nu - \varepsilon \notin T_\varphi \quad \text{for any} \ \ \varepsilon \in \mathbb{R}^n_+\}.$$

Let G be a sectorial set

$$G = \bigcup_{\nu \in \mathcal{M}_\varphi} \{z \in \mathbb{C}^n : \nu_j < \arg z_j < 2\pi - \nu_j, \ j = 1, ..., n\}. \tag{2}$$

Theorem. *Let $\varphi(\zeta)$ be an entire function of the exponential type interpolating coefficients f_k of the series (1). If there is $\nu(\theta) \in \mathcal{M}_\varphi(\theta)$ for $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]^n$ satisfying the inequalities*

$$\nu_j(\theta) \le a|\sin\theta_j| + b\cos\theta_j, \quad j = 1, ..., n,$$

where $a \in [0,\pi)$, $b \in [0,\infty)$, then the sum of the series extends analytically to a sectorial domain G of the form (2).

Idempotent Fourier multipliers acting contractively on H^p spaces

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I will present a joint work with Ole Fredrik Brevig (Oslo University) and Kristian Seip (Trondheim University). We describe the idempotent Fourier multipliers that act contractively on $H^p(\mathbb{T}^d)$. When p is not an even integer, such multipliers are just restrictions of contractive idempotent multipliers on $L^p(\mathbb{T}^d)$ spaces, which in turn can be described by suitably combining results of Rudin and Andô. When $p = 2(n+1)$, contractivity depends in an interesting geometric way on n, d, and the dimension of the set of frequencies associated with the multiplier. Our results allow us to construct a linear operator that is densely defined on $H^p(\mathbb{T}^\infty)$ for every $1 \le p \le \infty$ and that extends to a bounded operator if and only if $p = 2, 4, \ldots, 2(n+1)$.

On zeroes and poles of Helson zeta function

Roman Romanov

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The structure of poles and zeroes of the Helson zeta function, $\zeta_\chi(s) = \sum_1^\infty \chi(n)n^{-s}$, is studied. In particular, it is shown that two arbitrary disjoint sets in the critical strip $21/40 < \operatorname{Re} s < 1$ not accumulating off the left boundary $\operatorname{Re} s = 21/40$ are the sets of zeroes and poles of ζ_χ, respectively, for an appropriate choice of the completely multiplicative unimodular function χ.

This is a joint work with I. Bochkov.

A direct proof of Stahl's theorem for a generic class of algebraic functions

Sergei Suetin

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Under the assumption of the existence of Stahl's S-compact set, we give a short proof of the limit zero distribution of Padé polynomials and convergence in capacity of diagonal Padé approximants for a generic class of algebraic functions. The proof is direct but not from the opposite as Stahl's original proof is. The generic class means in particular that all branch points of the multi-sheeted Riemann surface of the algebraic function are of the first order (i.e., we assume the surface is such that all branch points are of square root type).

We do not use the orthogonality relations at all. The proof is based on the maximum principle only.

Invertibility threshold for Nevanlinna quotient algebras

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Let $\mathcal{N}$ be the Nevanlinna class and let B be a Blaschke product. Consider the natural necessary condition for invertibility of $[f]$ in the quotient algebra $\mathcal{N}/B\mathcal{N}$: "$|f| \geq e^{-H}$ on the zero set of B, for some positive harmonic function H". For large enough functions H, this is almost a sufficient condition if and only if the function $-\log|B|$ has a harmonic majorant on the set $\{z \in \mathbb{D} : \rho(z, \Lambda) \geq e^{-H(z)}\}$.

We thus study the class of harmonic functions H such that this last condition holds, and give some examples of B where it can be entirely determined.

The measures with L^2-bounded Riesz transform and the Painlevé problem for Lipschitz harmonic functions

Xavier Tolsa

ICREA & Universitat Autonoma de Barcelona

In this talk I will explain a recent work, partially in collaboration with Damian Dabrowski, where we provide a geometric characterization of the measures μ in $\mathbb{R}^{n+1}$ with polynomial upper growth of degree n such that the Riesz transform $R\mu(x) = \int \frac{x-y}{|x-y|^{n+1}}\, d\mu(y)$ belongs to $L^2(\mu)$. As a corollary, we obtain a characterization of the removable sets for Lipschitz harmonic functions in terms of a metric-geometric potential and we deduce that the class of removable sets for Lipschitz harmonic functions is invariant by bilipschitz mappings.

From Krawtchouk to Hahn via Jacobi and some other applications of differential operators with polynomial coefficients

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Consider a linear differential operator of the form

$$\mathcal{L}_r u \overset{def}{=} \sum_{j=0}^{r} Q_j(z)\frac{d^j u(z)}{dz^j}, \tag{1}$$

where the coefficient Q_j is a polynomial of degree n_j, $j = 0, 1, \dots, r$, and $Q_0(0) = 0$. For certain choices of coefficients, the operator $\mathcal{L}_r$ has polynomial eigenfunctions. In particular, the infinite systems of classical orthogonal polynomials are eigenfunctions of certain specialisations of $\mathcal{L}_2$.

We are interested in describing operators of the form (1) that have at least one polynomial eigenfunction, as well as in studying their properties. The first goal of the talk is to present our approach to studying and recognising such operators. The second goal is to survey various known (finite and infinite) systems of polynomial eigenfunctions of operators of type (1) of order 1 and 2.

Additionally, we will present a variation of our method which turns out to be connected with the so-called tridiagonalisation of differential operators introduced [1] by E. Koelink and M. Ismail. In particular, we reveal an interesting relation between the Krawtchouk and Hahn polynomials relying on certain properties of the Jacobi polynomials.

The talk is based on a joint work with Alexander Dyachenko.

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On geometry of the unit ball of Paley–Wiener space over two intervals

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Given a Banach space X, denote by $b(X)$ its unit ball, $b(X) := \{f \in X, \|f\| \le 1\}$. An element $f \in X$ is called an *extreme point* of $b(X)$, if it is not a proper convex combination of two distinct points of $b(X)$. An element f in $b(X)$ is an *exposed point* of $b(X)$, if there exists a functional $\phi \in X^*$ such that $\|\phi\| = 1$ and the set $\{g \in X : \phi(g) = 1\}$ consists of one element, f. There is a large number of papers studying extreme and exposed points in different function spaces. In particular, the classical theorem of K. de Leeuw and W. Rudin (1958) states that the extreme points of the unit ball of the Hardy space H^1 on the unit disk are precisely the outer functions $f \in H^1$ with $\|f\|_1 = 1$. On the other hand, no description of the exposed points of $b(H^1)$ is known.

Let S be a compact set on the real line. Denote by PW^1_S the space of integrable functions on the real line whose Fourier transform vanishes outside S equipped with the L^1-norm. We are interested in the following

Problem (K. Dyakonov, 2021). Describe the sets of extreme and exposed points of $b(PW^1_S)$.

When $S = [-\sigma, \sigma], \sigma > 0$, is a single interval, a complete solution of this problem was obtained by K. Dyakonov in 2000. In particular, a function f is an extreme point of $b(PW^1_S)$ if and only if $\|f\|_1 = 1$, at least one of the points $\sigma, -\sigma$ lies in the (closed) spectrum of f and f has no pairs of zeros symmetric with respect to the real line.

We consider the spectra S which consist of two symmetric intervals,

$$S := [-\sigma, -\rho] \cup [\rho, \sigma] = [-\sigma, \sigma] \setminus (-\rho, \rho), \quad 0 < \rho < \sigma.$$

We say that S has the gap $(-\rho, \rho)$.

It turns out that the structure of the set of extreme and the set of exposed points of $b(PW^1_S)$ depends on the size of gap. If $\rho > \sigma/2$ ("long gap"), then the description of these sets is somewhat similar to the one given by K. Dyakonov for the case of single interval. However, if $0 < \rho < \sigma/2$ ("short gap"), then the structure of these sets becomes more complicated. For the case of long gap, an essential step of the proof is to show that the exponential system with frequencies at the symmetric zeros of f is not complete in the space L^2 on some proper subinterval of $(-\rho, \rho)$. To prove this, we use the classical Beurling–Malliavin completeness theorem and a recent result on density of sign changes of real measures with spectral gap at the origin by M. Mitkovski and A. Poltoratski.

The talk is based on joint work with Ilya Zlotnikov.

Strange property of positive measures and bi-linear estimates on multi-trees

Alexander Volberg

Michigan State University

Carleson embedding theorems often serve as a first building block for interpolation in complex domain, for the theory of Hankel operators and in PDE. The embedding of certain spaces of holomorphic functions on n-polydisc can be reduced (without loss of information) to the boundedness of weighted multi-parameter dyadic Carleson embedding. We find the necessary and sufficient condition for this Carleson embedding in n-parameter case, when n is 1, 2, or 3. The main tool is the harmonic analysis on graphs with cycles. The answer is quite unexpected and seemingly goes against the well known difference between box and Chang–Fefferman condition that was given by Carleson quilts example of 1974. The main tool is an unexpected combinatorial property of positive measures on cube in dimensions 1, 2, 3. I will present results obtained jointly by Arcozzi, Holmes, Mozolyako, Psaromiligkos, Zorin–Kranich and myself.

Schäffer's conjecture, Fourier coefficients of Blaschke products and Jacobi polynomials with first varying parameter

Rachid Zarouf

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We prove results that we found on our way to a deeper understanding of Schäffer's conjecture about inverse operators. In 1970 J.J. Schäffer proved that for any invertible $n \times n$ matrix T and for any operator norm $||\cdot||$, the inequality

$$|\det T| \cdot ||T^{-1}|| \leq \mathcal{S}\, ||T||^{n-1}$$

holds with $\mathcal{S} = \mathcal{S}(n) \leq \sqrt{en}$. He conjectured that in fact this inequality holds with an $\mathcal{S}$ independent of n. This conjecture was refuted in the early 90's by E. Gluskin, M. Meyer and A. Pajor who have shown that for certain $T = T(n)$ the inequality can only hold when $\mathcal{S}$ is growing with n. Subsequent contributions of J. Bourgain and H. Queffélec provided increasing lower estimates on Schäffer's $\mathcal{S}$. Those results rely on probabilistic and number theoretic arguments for the existence of sequences $T(n)$ with growing $\mathcal{S}$. Constructive counterexamples to Schäffer's conjecture were not available since 1995. In this talk we propose a new and entirely constructive approach to Schäffer's conjecture. As a result, we present an explicit sequence of Toeplitz matrices T_λ with singleton spectrum $\{\lambda\} \subset \mathbb{D}\setminus\{0\}$ such that $\mathcal{S} \geq c(\lambda)\sqrt{n}$. A key ingredient in our approach will be to study ℓ_p-norms of Fourier coefficients of powers of a Blaschke factor, which is an interesting and well-studied topic in its own right, initiated by J-P. Kahane in 1956. Finally, on our way, we prove new estimates for the asymptotic behaviour of Jacobi polynomials with varying parameters and we highlight some flaws in the established literature on this topic.

This is based on a joint work with Oleg Szehr.